Progression in Calculation

## <u>Addition</u>

| Objective  | Concrete  | Pictorial   | Abstract  |
|--|---|---|---|
| and  |   |   |   |
| Strategies   |   |   |   |
| Combining two<br>parts to make a<br>whole: part-<br>whole model.<br>Counting all<br>objects using<br>one-to-one<br>correspondence<br>. Teach part-<br>whole addition |   | Use pictures or colour<br>squares cubes in bar<br>model shape to add two  | 4 + 3 = 7 $10 = 6 + 4$ $3$ Use the part-part<br>whole diagram as  |
| and subtraction<br>alongside each<br>other.  | Use cubes to add two<br>numbers together as a group<br>or in a bar.   |   | shown above to<br>move into the<br>abstract.  |
| Use known<br>number facts  | Concrete resources to make addition<br>pairs/number bonds, see patterns and<br>memorise. Number bonds for all totals:<br>1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 20. | Draw resources, make<br>pictures<br>$\begin{array}{c} 3+2=1\\ 2+2=8\\ $ | <b>She storg of</b><br><b>3</b><br>1+2=<br>2+1=<br>3+0=<br><b>10</b><br>Fill in the gap: $8 = 4 + \Box$ |

|   | 20 = 1 + 19 2 + 18 3 + 17   |  | Describe and use patterns<br>and relationships in<br>calculations once number<br>bonds for 10 are known.          |
|---|---|--|---|
| Starting at the<br>bigger number<br>and counting on   | 9 GOGOOGOGOGO ()  | Counting on using bar model diagram. Draw<br>picture of cubes/bar.   | Place the larger number in<br>your head and count on<br>the smaller number to find                                |
| (should only<br>count on in<br>ones for small<br>quantities, then<br>use knowledge<br>of number<br>bonds on | Start with the larger number on the<br>bead string and then count on to find<br>the answer: "12 add 5: 12, 13, 14, 15, 16,<br>17. There are 17 altogether." | 4+5=9  | your answer.<br>5 + 12 = 17<br>If I put 3 cubes into the<br>bag, then I put 2 more in<br>how many are in the bag? |
| partitioning<br>into tens/ones)   | 3 add 2<br>3+2  | 12 + 5 = 17:<br>10 11 12 13 14 15 16 17 18 19 20   |   |
|   | "3 are in the bag. Count in 2 more:: 4, 5.<br>5 are in the bag now."  | Start at the larger number on the number line and count<br>on in ones or in one jump to find the answer. Draw on a<br>given number line or draw own number line. |   |



| 0 as a place holder. | Use resources to make multiples of     | Draw pictures of resources, e.g. Numicon, ten frames, | Look at patterns. What do |
|----------------------|--|---|---------------------------|
| Adding 'ones' to 10  | 10 (10, 20, 30, 40) to see that the 0  | bead strings.   | you notice:               |
| (on a multiple of    | is needed to show that the number is   |   | 10+1=11                   |
|                      | 10 not 1.                              |   | 10+2=12                   |
| Ten).                |  |   | 10+3=13                   |
|                      | Use resources and place value arrow    |   | 10+4=14                   |
|                      | cards to add a single digit number (1- |   |                           |
|                      | 9) to 10 or a multiple of 10:          |   | How is the                |
|                      |  |   | same/difference to adding |
|                      |  |   | to 202                    |
|                      | 60                                     |   | 20+1=21                   |
|                      |  |   | 20+2=22                   |
|                      |  |   | 20+3=23                   |
|                      |  |   |                           |
|                      |  |   | What is the missing       |
|                      |  |   | number?                   |
|                      | 10+5: put them together to make 15.    |   | 20+□=22                   |
|                      | The 5 goes on top of the 0. The 0 is   |   |                           |
|                      | keeping the space/place open           |   | 24 = 20 + 1               |
|                      |  |   |                           |
|                      |  |   |                           |
|                      | 10+2=12                                |   |                           |
| Adding three         | 4 + 7 + 6= 17                          | a a all a   |                           |
| sinale diaits (first | Put 4 and 6 together to make 10. Add   |   |                           |
| look for two         | on 7.                                  |   | (4) + 7 + 6 = 10 + 7      |
| numbers that make    | Combine esses                          | Regroup and draw representation                       | 10                        |
| 10)                  | two                                    | Regioup and draw representation.                      | = 17                      |
| 10)                  | arouns to                              |   | Combine the two numbers   |
|                      | make 10 and then add the third         |   | that make 10 and then add |
|                      | aroup.                                 | <b>2</b> + <b>2</b> = 15                              | on the remainder          |
|                      | J                                      | Add together three groups of phiests Draws            |                           |
|                      |  | nicture to recombine the crowned to make 10           |                           |
|                      |  | picture to recombine the groups to make 10.           |                           |
|                      |  |   |                           |





<u>Subtraction</u>

| Objective and       | Concrete                                | Pictorial   | Abstract                |
|---------------------|---|---|-------------------------|
| Strategies          |   |   |                         |
| Taking away ones    | Use physical objects, counters, cubes   | Cross out drawn objects to show what has been taken | 18 -3= 15               |
| (at first place     | etc to show how objects can be taken    | away. The bar model can also be used.               |                         |
| objects on top of   | away.<br>4 - 3 = 1                      | * * * * *   | 8 - 2 = 6               |
| any problems        |   |   | What is 12 take away 52 |
| presented in        |   | <u> </u>  |                         |
| pictures to ensure  |   | * * * * *   | 6=8-2                   |
| concrete            |   |   |                         |
| and enable          |   | 15 - 3 = 12   |                         |
| children to move    | ALL |   | Part Relate to          |
| objects/take away   |   |   | whole part-             |
| and count with 1 to | Fingers                                 |   | part 5.2-2 and          |
| 1 correspondence)   | a martine and a second                  |   | 5-2=3                   |
|                     |   |   |                         |
|                     | 6 - 2 = 4                               |   |                         |
|                     |   |   | 4-3=                    |
|                     |   |   | 1770                    |
|                     |   |   | = 4 - 5                 |
|                     |   |   | 4                       |
|                     | a                                       |   | 3 ?                     |
|                     | 14 - 5                                  |   |                         |
|                     |   |   |                         |
|                     |   |   |                         |
|                     |   |   |                         |
|                     |   |   |                         |
|                     |   |   |                         |



|  | Counting back using objects and a<br>number track. Start at 6 and count<br>back 2:<br>6 - 2 = 4<br>1 2 3 4 5 6 7 8 9 10  |  |   |
|--|--|--|---|
| Find the<br>difference<br>(understanding<br>finding the<br>difference as<br>subtraction,<br>vocab: how many<br>more/how many<br>fewer, what is<br>the difference | Compare amounts and objects to find<br>the difference.<br>Use cubes to build towers or make<br>bars to find the difference<br>Use basic bar models with items to<br>find the difference<br>Which line has most money | Draw draw lines of<br>objects, moving onto<br>drawing boxes around<br>pictures and later on<br>draw bars to find<br>the difference between 2<br>numbers.<br>Comparison Bar Models<br>Lisa is 13 years old. Her sister is 22 years old.<br>Find the difference in age between them. | Hannah has 23<br>sandwiches, Helen has 15<br>sandwiches. Find the<br>difference between the<br>number of sandwiches.<br>How much more is 12 than<br>8? How much less is 10<br>than 15?<br>20-18=2 |
| etc).  | How much more?   | Lisa<br>Sister   | 6-4=2<br>The difference between 6<br>and 4 is 2.  |
| Counting up to<br>calculate<br>subtraction<br>especially when<br>difference is<br>small (relate to<br>finding the difference,<br>part part whole facts)          | See above  | Same difference: whether you count up or back the answer is the same:<br>The difference between II and I4 is 3.<br>It is quicker and more efficient to count up 3 jumps than jump back 11 jumps.   | The difference<br>between II<br>and I4 is 3.<br>I4 - II = 3<br>$II + \Box = I4$<br>11<br>3  |

## Part Part Whole Model



Link to addition- use the part whole model to help explain the inverse between addition and subtraction. Use a pictorial representation of objects to show the part part whole model and work out the answer.



The whole is 8. The parts are 4 and 4. 8-4=4.



The whole is . The parts are 2 and 7

| 5   |
|---|
| Move to using numbers<br>within the part whole<br>model.                    |
| If 10 is the whole and 6 is<br>one of the parts. What is<br>the other part? |
| 10 - 6 = □<br>10 - □ = 5<br>□ - 3 = 7                                       |
| I think of a number, I add<br>2. My answer is 10. What<br>did I start with? |
| Use numbers to represent the part part whole                                |
| 8   |
| 6 2   |

| Use known<br>number facts<br>(use inverse,<br>and derive)  | As above: use addition number facts<br>to help with subtraction number<br>facts. Use part whole diagram and<br>resources.<br>Use resources to help see how facts<br>can be derived. E.g. 8-4=4 made with<br>dienes ones. 80-40=40 using dienes<br>tens. | Draw part whole diagram and resources to help use<br>addition facts to work out subtraction facts<br>Draw dienes ones to show<br>8-4=4<br>Draw dienes tens to show<br>80-40=40 | If I know 8-4=4, what<br>other subtraction facts do<br>I know?                    |
|--|---|--|---|
| Regroup a 10<br>into ten ones<br>(useful to<br>support in<br>mental<br>calculation<br>within 20) | 20-4 with Dienes or Numicon. Will<br>need to regroup/exchange the 10 for<br>ten ones.   | Draw resources. Draw pictures.   | 20-4=16   |
| Make<br>10/bridging<br>through 10  | 14 - 5 =<br>Make 14 on the ten frame. Take away<br>the four first to make 10 and then<br>takeaway one more so you have taken  | Draw resources used. Draw given number on a tens<br>frame, number line or draw own number line.  | $ \begin{array}{c} 14 - 5 = 9 \\ 4 & 1 \\ 14 - 4 = 10 \\ 10 - 1 = 9 \end{array} $ |

|   | away 5. You are left with the answer<br>of 9.<br>42-15<br>E.g. partition 15 into 2 and 10 and 3<br>to subtract.                               | 13 - 7 = 6<br>3 4<br>5 + 2 + 3 + 4 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5   |   |
|---|---|--|---|
| Subtract<br>multiples of 10<br>from a 2 digit<br>number         | $\frac{1}{23-10}$ Aake with resources. Notice what happens to the tens and the ones $63 - 40$ $\frac{1}{100}$ $\frac{1}{100}$ $\frac{1}{100}$ | Draw resources/work with pictures  | 23-10=13<br>Make connections and<br>describe patterns:<br>43-10=33<br>33-10=23<br>23-10=13<br>13-10=3 |
| Subtraction of<br>two digit<br>numbers<br>without<br>regrouping | Tens Ones Use Base<br>10/Dienes<br>to make the<br>bigger<br>number then<br>take the<br>smaller  | 10       ●       76-64=12         10       ●       < | 47-22:<br>7-2=5<br>40-20=20<br>Recombine: 20+5=25   |

|   | number away.  | 43-21 = 22<br>Draw the Dienes or place value counters alongside the written calculation to help to show working.   |           |
|---|---|--|-----------|
| Subtraction of<br>two digit<br>numbers with<br>regrouping | Children use concrete resources to<br>practically exchange 1 ten for ten<br>ones<br>41-26<br>10s 1s<br>10s 1s<br>10s 1s<br>1<br>5 | Children to represent subtraction with regrouping.<br>10s 1s $11s$ | 45 - 23 = |

| Subtracting     | _        |                             | 38-10=28   |
|-----------------|----------|-----------------------------|--|
| multiples of 10 |          |                             | 40=60-20   |
|                 |          | Draw resources or pictures. | Make connections and<br>describe patterns:<br>If I know 8-4=4, then I<br>know that<br>80-40=40 |
|                 | 60-20=40 |                             |  |

## **Multiplication**

| Objective and            | Concrete   | Pictorial   | Abstract  |
|--------------------------|--|---|---|
| Strategies               |  |   |   |
| Doubling                 | double 4 is 8<br>$4 \times 2 = 8$<br>Use practical activities to show how<br>to double a number. | Draw pictures to show how to double a number.<br>Double 4 is 8              | 16<br>10<br>10<br>10<br>10<br>10<br>10<br>10<br>10<br>10<br>10  |
| Counting in<br>multiples | Concrete objects in equal groups.  | Use a number line or pictures to continue support in counting in multiples. | Count in multiples of a<br>number aloud.<br>Write sequences with<br>multiples of numbers.<br>2, 4, 6, 8, 10<br>5, 10, 15, 20, 25 , 30 |







<u>Division</u>

| Objective and  | Concrete  | Pictorial   | Abstract   |
|--|---|---|--|
| Sharing objects<br>into groups   | 6÷2   | Children use pictures or shapes to share quantities.<br>$6 \div 2$<br>$6 \div 2$<br>$6 \div 2 = 3$<br>$6 \div 2 = 3$  | Share 9 buns between<br>three people.<br>9 ÷ 3 = 3                       |
| Division as<br>grouping (can<br>also relate to<br>multiplication<br>facts) | Divide quantities into equal groups<br>(repeated subtraction).<br>Use cubes, counters, objects or place<br>value counters to aid understanding. | Draw counters and identify equal groups<br>Use a number line to show repeated subtraction of<br>groups. The number of jumps equals the number of<br>groups. 12÷3=4 Relate to 3x table. 3x4=12 | 28 ÷ 7 = 4<br>Divide 28 into 7 groups.<br>How many are in each<br>group? |

|  | 0       5       10       15       20       25       30       35         35 grouped into fives makes 7 groups.       35÷5=7         8 ÷ 4 =       Image: Complexity of the second seco | 0 1 2 3 4 5 6 7 8 9 10 11 12<br>3 3 3 3 3<br>Think of the bar as a whole. Split it into the number<br>of groups you are dividing by and work out how many<br>would be within each group using times table facts.<br>20<br>20 $\div$ 5 = ?<br>5 x ? = 20 |   |
|--|--|---|---|
| Division within<br>arrays (inverse<br>relationship<br>with<br>multiplication;<br>understanding<br>that x is<br>commutative<br>but division is<br>not). | Link division<br>to<br>to<br>to<br>to<br>to<br>to<br>to<br>to<br>to<br>to<br>to<br>to<br>to  | - () () () () () () () () () () () () ()  | Find the inverse of<br>multiplication and division<br>sentences by creating four<br>linking number sentences.<br>7 × 4 = 28<br>4 × 7 = 28<br>28 ÷ 7 = 4<br>28 ÷ 4 = 7 |

| Partitioning/<br>regrouping to<br>divide   | Dienes, numicon etc.<br>Look for 'numbers inside numbers' and<br>partition tens into ones. 30 cannot<br>be 'shared' into 2, or ÷2 when it is<br>represented as 3 Dienes tens. If a<br>child knows that 20 can be ÷2 they<br>can partition 30 into 20 and 10, and<br>calculate 20÷2 and 10÷2 then<br>recombine. | Draw resources   | $36+2 \\ \rightarrow \\ 30 \ 6 \ 6+2=3 \\ 20 \ 10 \\ \psi \\ 20+2=10 \\ 10+2=5 \\ \psi \\ 10+5+3=18 \\ $       |
|--|--|--|--|
| Division with a  | 14 ÷ 3 =   | Jump forward in equal jumps on a number line then  | Complete written divisions   |
| remainder  | Divide objects between groups<br>and see how much is left over   | 0 4 8 12 13<br>see how many more you need to jump to find a<br>remainder.<br>Draw dots and group them to divide an amount and<br>clearly show a remainder. | and show the remainder<br>using r.<br>29 ÷ 8 = 3 REMAINDER 5<br>↑ ↑ ↑ ↑<br>dividend divisor quotient remainder |
| Fractions:<br>relating finding<br>$\frac{1}{2}$ by dividing by<br>2, and $\frac{1}{4}$ by<br>dividing by 4 | Half of 8 is 4   | Draw resources<br>Bar Model<br>1 thrd of<br>18   | ½ of 26 = 13<br>26 ÷ 2 = 13  |



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